# Sectioned Warp Shell Dynamics (Upgraded): Toroidal Vortex-Canon Sources, Explicit Section/Antisection Basis, and Control Topology

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*Simulonic Research Notes*

**Abstract**

We upgrade the sectioned ellipsoidal warp-shell model by (i) replacing generic actuation sources with an explicit dual Vortex-Canon construction that generates counter-rotating toroidal rings, and (ii) defining warp sections/antisections as explicit smooth basis functions on an ellipsoidal shell. A scalar warp potential is driven by toroidally-induced source terms derived from vorticity-tube kernels. Twelve Simulonic Field Generators (SFGs) on a midship toroidal ring implement phase exchange between opposing antisections via a discrete phase-coupled envelope law. A Harmonic Oscillator (HO) supplies a global phase reference; an Inertial Null Core (INC) imposes an inertial-gradient sink.

*Keywords:* Vortex Canon, Toroidal Rings, Warp Sections, Warp Antisections, SFG Lattice, HO Phase Lock, INC Sink, Control Topology

## 1. Geometry: Ellipsoidal Shell and Coordinates

Let the warp shell be an ellipsoid centered on the craft axis $\hat{\mathbf{x}}$:

$$\boxed{\partial\Omega \;:=\; \left\{(x,y,z)\in\mathbb{R}^3 :\; \frac{x^2}{c^2}+\frac{y^2+z^2}{a^2}=1\right\}, \qquad c>a>0.} \tag{1}$$

Define *ellipsoidal angular coordinates* $(\vartheta,\varphi)$ by:

$$\boxed{\mathbf{s}(\vartheta,\varphi)=\Big(c\cos\vartheta,\; a\sin\vartheta\cos\varphi,\; a\sin\vartheta\sin\varphi\Big), \quad \vartheta\in[0,\pi],\; \varphi\in[0,2\pi).} \tag{2}$$

Here $\vartheta$ is axial (anterior/posterior) and $\varphi$ is azimuthal (dorsal/ventral and port/starboard).

## 2. Field Variables and Baseline Warp Potential Model

Introduce a scalar warp potential $\Phi(\mathbf{x},t)$ over the shell interior $\Omega$, with flux-like field

$$\boxed{\mathbf{J}(\mathbf{x},t) \;:=\; -\kappa\,\nabla\Phi(\mathbf{x},t), \qquad \kappa>0.} \tag{3}$$

We use a damped screened-wave form:

$$\boxed{\partial_{tt}\Phi+2\zeta\omega_0\,\partial_t\Phi=c_\Phi^2\,\nabla^2\Phi-\omega_0^2\,\Phi+S_{\mathrm{VC}}(\mathbf{x},t)+S_{\mathrm{SFG}}(\mathbf{x},t).} \tag{4}$$

MEA strips act as boundary smoothing/damping on regions $\Gamma_A,\Gamma_P\subset\partial\Omega$:

$$\boxed{\alpha\,\Phi+\beta\,\partial_n\Phi=b_{\mathrm{MEA}}(\mathbf{x},t), \qquad \mathbf{x}\in\Gamma_A\cup\Gamma_P.} \tag{5}$$

## 3. Upgrade I: Vortex-Canon Toroidal Ring Source

Let the ring station be $x=x_c$ (midship), major radius $R_c$, core radius $\delta$. Define cylindrical radius about $\hat{\mathbf{x}}$:

$$\rho(\mathbf{x})=\sqrt{y^2+z^2}.$$

Two Vortex Canons generate opposed circulation:

$$\Gamma_+=+\Gamma, \qquad \Gamma_-=-\Gamma, \qquad \Gamma>0.$$

A toroidal vorticity-tube kernel (about $\hat{\mathbf{x}}$) is:

$$\boxed{\boldsymbol{\omega}_i(\mathbf{x})=\Gamma_i\,\exp\!\left(-\frac{(\rho(\mathbf{x})-R_c)^2+(x-x_c)^2}{2\delta^2}\right)\hat{\boldsymbol{\phi}}(\mathbf{x}), \qquad i\in\{+,-\}.} \tag{6}$$

Total vorticity: $\boldsymbol{\omega}=\boldsymbol{\omega}_+ + \boldsymbol{\omega}_-$.

### 3.1. Induced flux analog (Biot–Savart form)

$$\boxed{\mathbf{u}(\mathbf{x})=\frac{1}{4\pi}\int_{\mathbb{R}^3}\frac{\boldsymbol{\omega}(\mathbf{x}')\times(\mathbf{x}-\mathbf{x}')}{\|\mathbf{x}-\mathbf{x}'\|^3}\,\mathrm{d}^3x'.} \tag{7}$$

Inject into the warp potential via:

$$\boxed{S_{\mathrm{VC}}(\mathbf{x},t):=\lambda_{\mathrm{VC}}\,\nabla\cdot\mathbf{u}(\mathbf{x},t)\;+\;\lambda_{\mathrm{tw}}\,\mathcal{T}(\mathbf{x},t).} \tag{8}$$

A minimal twist/interaction term:

$$\boxed{\mathcal{T}(\mathbf{x},t)=-\Gamma^2\;\exp\!\left(-\frac{(\rho(\mathbf{x})-R_c)^2+(x-x_c)^2}{\delta^2}\right)\Xi(t),} \tag{9}$$

where $\Xi(t)$ is a modulation factor.

## 4. Upgrade II: Explicit Section/Antisection Basis on the Shell

Define a smooth step and window:

$$\sigma(x;k) = \frac{1}{1+e^{-kx}}, \qquad k > 0,$$

$$\boxed{W_{[p,q]}(\xi;k) := \sigma(\xi - p;k)\,\sigma(q-\xi;k).} \tag{10}$$

### 4.1. Four warp sections (I–IV)

Choose two axial bands:

$$\vartheta \in [\vartheta_A^-, \vartheta_A^+] \quad \text{(anterior band)}, \qquad \vartheta \in [\vartheta_P^-, \vartheta_P^+] \quad \text{(posterior band)},$$

and azimuthal halves:

$$\varphi \in [0,\pi), \qquad \varphi \in [\pi, 2\pi).$$

$$\boxed{\psi_1(\vartheta,\varphi) := W_{[\vartheta_A^-,\vartheta_A^+]}(\vartheta;k)\,W_{[0,\pi]}(\varphi;k)} \tag{11}$$

$$\boxed{\psi_2(\vartheta,\varphi) := W_{[\vartheta_A^-,\vartheta_A^+]}(\vartheta;k)\,W_{[\pi,2\pi]}(\varphi;k)} \tag{12}$$

$$\boxed{\psi_3(\vartheta,\varphi) := W_{[\vartheta_P^-,\vartheta_P^+]}(\vartheta;k)\,W_{[0,\pi]}(\varphi;k)} \tag{13}$$

$$\boxed{\psi_4(\vartheta,\varphi) := W_{[\vartheta_P^-,\vartheta_P^+]}(\vartheta;k)\,W_{[\pi,2\pi]}(\varphi;k)} \tag{14}$$

### 4.2. Four warp antisections (i–iv)

Choose endcap bands:

$$\vartheta \in [0,\vartheta_F] \quad \text{(front cap)}, \qquad \vartheta \in [\pi-\vartheta_R,\pi] \quad \text{(rear cap)}.$$

Choose dorsal/ventral windows with half-width $\Delta_\varphi$:

$$W_D(\varphi) := W_{[\frac{\pi}{2}-\Delta_\varphi,\frac{\pi}{2}+\Delta_\varphi]}(\varphi;k), \quad W_V(\varphi) := W_{[\frac{3\pi}{2}-\Delta_\varphi,\frac{3\pi}{2}+\Delta_\varphi]}(\varphi;k).$$

$$\boxed{\bar\psi_1(\vartheta,\varphi) := W_{[0,\vartheta_F]}(\vartheta;k)\,W_D(\varphi)} \tag{15}$$

$$\boxed{\bar\psi_2(\vartheta,\varphi) := W_{[0,\vartheta_F]}(\vartheta;k)\,W_V(\varphi)} \tag{16}$$

$$\boxed{\bar\psi_3(\vartheta,\varphi) := W_{[\pi-\vartheta_R,\pi]}(\vartheta;k)\,W_D(\varphi)} \tag{17}$$

$$\boxed{\bar\psi_4(\vartheta,\varphi) := W_{[\pi-\vartheta_R,\pi]}(\vartheta;k)\,W_V(\varphi)} \tag{18}$$

### 4.3. Boundary expansion

$$\boxed{\Phi\big|_{\partial\Omega}(\vartheta,\varphi,t) = \sum_{j=1}^{4} a_j(t)\,\psi_j(\vartheta,\varphi) - \sum_{\ell=1}^{4} \bar a_\ell(t)\,\bar\psi_\ell(\vartheta,\varphi).} \tag{19}$$

## 5. State Estimation: Extracting Section/Antisection Amplitudes

Let $\mathrm{d}A$ be the surface area element on $\partial\Omega$ (from (2); numerically approximated). Normalization constants:

$$Z_j = \int_{\partial\Omega} \psi_j(\vartheta,\varphi)\,\mathrm{d}A, \qquad \bar Z_\ell = \int_{\partial\Omega} \bar\psi_\ell(\vartheta,\varphi)\,\mathrm{d}A.$$

Estimated amplitudes:

$$\boxed{\hat a_j(t) = \frac{1}{Z_j}\int_{\partial\Omega} \Phi(\mathbf{s}(\vartheta,\varphi),t)\,\psi_j(\vartheta,\varphi)\,\mathrm{d}A} \tag{20}$$

$$\boxed{\hat{\bar a}_\ell(t) = \frac{1}{\bar Z_\ell}\int_{\partial\Omega} \Phi(\mathbf{s}(\vartheta,\varphi),t)\,\bar\psi_\ell(\vartheta,\varphi)\,\mathrm{d}A} \tag{21}$$

## 6. SFG Toroidal Lattice: Discrete Actuation and Phase Exchange

Let $N = 12$ SFGs be on the ring:

$$\theta_k = \frac{2\pi k}{12},\ k = 0,\dots,11, \qquad \mathbf{p}_k = (x_c,\ R_c\cos\theta_k,\ R_c\sin\theta_k).$$

$$\boxed{S_{\mathrm{SFG}}(\mathbf{x},t) = \sum_{k=0}^{11} u_k(t)\,G_\sigma(\mathbf{x}-\mathbf{p}_k), \qquad G_\sigma(\mathbf{r}) = \exp\!\left(-\frac{\|\mathbf{r}\|^2}{2\sigma^2}\right).} \tag{22}$$

HO reference and phase-lock:

$$\theta(t) = \omega_{\mathrm{HO}}t + \phi_0, \qquad u_k(t) = \mathrm{Re}\{U_k(t)e^{i\theta(t)}\}.$$

Opposition map:

$$\pi(1) = 3,\ \pi(3) = 1, \qquad \pi(2) = 4,\ \pi(4) = 2.$$

Antisection exchange:

$$\boxed{\dot{\bar a}_\ell = -\gamma\,\bar a_\ell + k_\ell\,(\bar a_{\pi(\ell)} - \bar a_\ell) + \upsilon_\ell(t).} \tag{23}$$

Partition SFGs into four groups $\mathcal{K}_\ell$ (three each), define group envelopes:

$$U_\ell(t) = \frac{1}{3}\sum_{k\in\mathcal{K}_\ell} U_k(t).$$

Phase-coupled drive:

$$\boxed{\upsilon_\ell(t) = \mu\ \mathrm{Im}\left\{U_\ell(t)\,\overline{U_{\pi(\ell)}(t)}\right\}.} \tag{24}$$

## 7. INC Sink and MEA Damping

INC sink constraint (conceptual):

$$\nabla\Phi(\mathbf{x},t) \to \mathbf{0} \quad \text{as} \quad \mathbf{x} \to \mathbf{p}_{\mathrm{INC}}.$$

MEA damping policy:

$$b_{\mathrm{MEA}}(\mathbf{x},t) = -\lambda_{\mathrm{MEA}}\,\Phi(\mathbf{x},t) - \nu_{\mathrm{MEA}}\,\partial_t\Phi(\mathbf{x},t), \quad \mathbf{x} \in \Gamma_A \cup \Gamma_P.$$

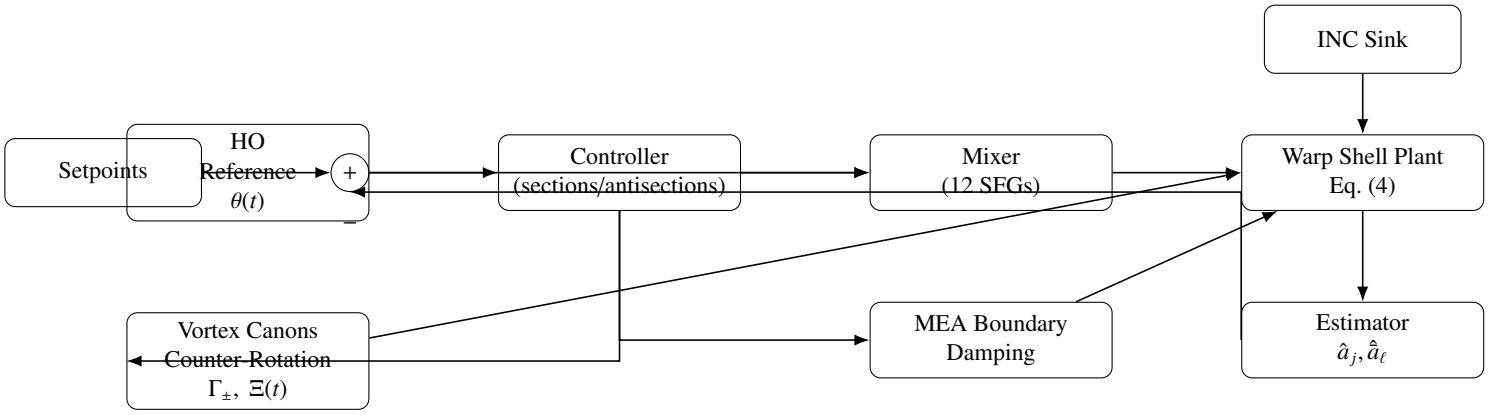


Figure 1: Upgraded closed-loop topology with explicit Vortex-Canon drive and section/antisection estimation using the basis functions on the ellipsoidal shell.

## 8. Control Topology Diagram

## 9. Conclusion

We removed generic actuation in favor of an explicit dual Vortex-Canon construction, and defined explicit smooth basis functions for sections and antisections on an ellipsoidal shell. This makes state estimation, phase exchange, and control allocation unambiguous and directly aligned to the four schematic depictions.