Simulonic Formalization: Counter-Rotating Toroidal Vortex Canons Generating an Ellipsoidal Warp Bubble

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Abstract

This document formalizes the construction: a *Vortex Canon* generates a toroidal ring in one direction; another Vortex Canon generates a toroidal ring in the opposing direction; together they produce an elongated (prolate) ellipsoid oval warp bubble. The model is written as a fluid/field analog using toroidal vorticity tubes, an induced vector field via Biot–Savart, and an iso-surface scalar "bubble intensity" field whose boundary approximates a prolate ellipsoid.

1 Coordinates and the Target Bubble

Let the craft axis be the z-axis.

Define a **prolate ellipsoid (elongated oval)** as the bubble boundary:

$$\boxed{\mathcal{B} = \left\{ (x,y,z) \in \mathbb{R}^3 : \ \frac{x^2+y^2}{a^2} + \frac{z^2}{c^2} = 1 \right\}, \quad c > a > 0} \tag{1}$$

If you want the bubble to move with the craft, shift $z \to z - z_0(t)$.

Define cylindrical radius:

$$\rho = \sqrt{x^2+y^2}. \tag{2}$$

2 Model Each "Vortex Canon" as a Toroidal Vorticity Ring

A vortex ring (toroidal ring) centered around the z-axis with major radius R and core radius δ can be represented by a **vorticity distribution** concentrated near the circle $(\rho = R, z = z_i)$.

Let $i \in \{+,-\}$ denote the two canons, located at $z = z_+$ and $z = z_-$, generating **opposite circulation** $\pm\Gamma$.

A compact analytic model for the vorticity (direction around the ring) is:

$$\boxed{\boldsymbol{\omega}_i(\rho,z) = \Gamma_i \, \exp\!\left(-\frac{(\rho - R)^2 + (z - z_i)^2}{2\delta^2} \right) \hat{\boldsymbol{\phi}}} \tag{3}$$

where

- $\hat{\boldsymbol{\phi}}$ is the azimuthal unit vector (toroidal direction),
- $\Gamma_+ = +\Gamma$, $\Gamma_- = -\Gamma$ (counter-rotation),
- R is the ring radius, δ is the thickness of the torus core.

This is the "toroidal ring in one direction / opposing direction" statement, formalized.

3 The Induced "Field-Flow" from Vorticity (Biot–Savart / Vector Potential Form)

In a fluid analog, velocity $\mathbf{u}$ is induced by vorticity via a Biot–Savart-type relation:

$$\boxed{\mathbf{u}(\mathbf{x}) = \frac{1}{4\pi}\int_{\mathbb{R}^3} \frac{\boldsymbol{\omega}(\mathbf{x}') \times (\mathbf{x}-\mathbf{x}')}{\|\mathbf{x}-\mathbf{x}'\|^3}\, d^3x'} \tag{4}$$

with $\boldsymbol{\omega} = \boldsymbol{\omega}_+ + \boldsymbol{\omega}_-$.

Interpretation for your system: $\mathbf{u}$ can stand in for an effective "warp field flux," "frame-drag flow," or whatever coupling field you like—mathematically it's a vector field generated by the two toroidal vorticity tubes.

4 Construct a "Bubble Intensity" Scalar Whose Iso-Surface Is Ellipsoidal

To encode "together they create an ellipsoid oval bubble," define a scalar $\Phi(\rho, z)$ built from **two counter-rotating rings**. A simple and very controllable way is:

4.1 Ring Kernels (Gaussian Distance to Each Torus)

Distance-from-torus (approx) for ring i:

$$d_i^2(\rho, z) = (\rho - R)^2 + (z - z_i)^2. \tag{5}$$

Define each ring's contribution:

$$\Phi_i(\rho, z) = A_i \exp\left(-\frac{d_i^2(\rho, z)}{2\delta^2}\right). \tag{6}$$

4.2 Counter-Rotation Enters as a Signed "Twist" Term

Counter-rotation doesn't change a purely positive magnitude field, so include a **signed coupling** term that depends on the product of circulations:

$$\boxed{\Phi(\rho, z) = \Phi_+(\rho, z) + \Phi_-(\rho, z) + \lambda\,(\Gamma_+\Gamma_-)\,\Phi_+(\rho, z)\Phi_-(\rho, z)} \tag{7}$$

Since $\Gamma_+\Gamma_- = -\Gamma^2 < 0$, the interaction term **subtracts** (or flips sign) in the overlap region. That overlap "pinches" and shapes the combined iso-surface into an elongated oval, rather than two separate donuts.

The **warp bubble boundary** is then an iso-surface:

$$\boxed{\partial\mathcal{W} = \{\mathbf{x} :\ \Phi(\mathbf{x}) = \Phi_0\}} \tag{8}$$

5 Enforce "Ellipsoid" Explicitly (Fit Parameters)

If you want the resulting iso-surface to match the ellipsoid $\frac{\rho^2}{a^2} + \frac{z^2}{c^2} = 1$, you can choose parameters to do it:

- Place rings symmetrically: $z_{\pm} = \pm z_0$.
- Choose the isovalue Φ_0 and solve for the best a, c by fitting $\partial\mathcal{W}$ to the ellipsoid (least squares on sampled points).
- A useful heuristic is:

$$c \approx z_0 + \kappa\delta, \qquad a \approx R + \kappa\delta \tag{9}$$

where κ depends on the chosen Φ_0.

6 "Formal Math Statement" Summary

Two Vortex Canons ⇒ Two Toroidal Vorticity Tubes

$$\boldsymbol{\omega}(\rho, z) = \Gamma\, e^{-\frac{(\rho-R)^2+(z-z_0)^2}{2\delta^2}}\,\hat{\boldsymbol{\phi}} - \Gamma\, e^{-\frac{(\rho-R)^2+(z+z_0)^2}{2\delta^2}}\,\hat{\boldsymbol{\phi}} \tag{10}$$

Induced Effective Flux/Flow Field (Biot–Savart)

$$\mathbf{u}(\mathbf{x}) = \frac{1}{4\pi}\int \frac{\boldsymbol{\omega}(\mathbf{x}') \times (\mathbf{x} - \mathbf{x}')}{\|\mathbf{x} - \mathbf{x}'\|^3}\, d^3x' \tag{11}$$

Bubble Intensity Field from Ring Overlap + Counter-Rotation Coupling

$$\Phi(\rho, z) = Ae^{-\frac{(\rho-R)^2+(z-z_0)^2}{2\delta^2}} + Ae^{-\frac{(\rho-R)^2+(z+z_0)^2}{2\delta^2}} - \lambda\Gamma^2\left(e^{-\frac{(\rho-R)^2+(z-z_0)^2}{2\delta^2}}\right)\left(e^{-\frac{(\rho-R)^2+(z+z_0)^2}{2\delta^2}}\right) \tag{12}$$

Warp Bubble Boundary as an Iso-Surface

$$\partial\mathcal{W} = \{\mathbf{x} : \Phi(\mathbf{x}) = \Phi_0\} \approx \left\{\mathbf{x} : \frac{x^2 + y^2}{a^2} + \frac{z^2}{c^2} = 1\right\} \tag{13}$$